REVIEWS

Introduction to Mechanics of Continua. By William Prager. Boston: Ginn and Co., 1961. 230 pp. $8.00.

This new book by Prof. Prager has a twofold aim stated in the preface: 'to provide the foundation for a deeper study of special branches such as hydrodynamics, gasdynamics, elasticity, and plasticity, and to present typical problems and methods of mechanics of continua for the benefit of those who do not plan to specialize in the field.' As can easily be imagined from this list of topics, there is barely space in a book of this size even to introduce the student to such a vast range of ideas and methods. Perforce, the author has had to call upon all his expository and organization skill to cover the ground. I think it is typical of all well-written introductions that they appear on first reading to hang together and form a well-rounded whole, but that careful scrutiny of any small part leaves the specialist unhappy and inclined to haggle. This book is not for specialists, and anyone who has faced the problem of organizing and teaching a senior or graduate level introductory course in continuum mechanics will appreciate Prof. Prager's achievement.

The book begins with an introductory chapter on vector and tensor analysis which seems to cover all that is needed in the sequel. This chapter is followed by brief chapters on stress and instantaneous motion. The latter term means what is more commonly called *steady motion,* but the chapter goes on to treat kinematics of time-dependent velocity fields as well. I found the terminology at this and other places in the book somewhat unusual. On the whole, these kinematical foundations, so important to all of continuum mechanics, seemed to me rather too sketchy even for an introductory treatment. There is no discussion of deformation and rotation until the final chapters. The author seems pleased to be able to develop the whole subject except for finite elasticity theory with no kinematical apparatus other than the rate of deformation and vorticity tensors. While it is true that by the device of introducing an infinitesimal displacement by writing $du_i(x) = v_i(x)\,dt$, one can in a formal way eliminate the mathematics of strain and rotation in the formulation of the constitutive relations of the classical linear theory of elasticity and derive them from hypoelasticity relations, this procedure does not do justice to the differences between the concepts of fluidity, elasticity and hypoelasticity.

This chapter on kinematics is followed by a chapter on the 'Fundamental Laws'. It is claimed to present the physical laws that apply to all continua. But we must caution that, for what we assume to be pedagogic reasons, the chapter contains numerous formulas which apply only to special materials and situations. For example, Fourier's constitutive relation for the heat flux, the internal energy of a perfect gas, and the stress-deformation relations of a linearly viscous fluid all appear in this chapter. Of course, to some it will be clear from the context what the author intends as the fundamental laws: namely, the principles of momentum, moment of momentum, and energy conservation. But I am afraid that, so placed, Fourier's law of heat conduction might be construed by the unwary beginner as sharing the stature of a basic principle of continuum mechanics. A small point is the author's use of the terminology 'momentum theorem', 'moment of momentum theorem', and 'energy theorem'. To be sure, these are the central principles or hypotheses of continuum mechanics, but, in a mathematical setting, a theorem is a statement that has been proved or conjectured as a consequence of previous assumptions.

There follows individual chapters on perfect fluids, viscous fluids, visco-plastic and perfectly plastic materials, and classical linear elasticity theory combined with hypoelasticity theory. In each of these chapters, the author does a remarkable job of introducing a vast number of special problems, methods, and typical solutions. References and guides to the literature are limited but chosen well. I am doubtful that there will be much agreement with Prof. Prager on the precise ordering of these special topics, especially in an introduction to continuum mechanics. To find the theory of visco-plastic materials preceding the classical linear theory of elasticity seems a bit strange, if not for reasons of difficulty but only for historical perspective. At any rate, this reviewer appreciated the clear and lucid chapter introducing visco-plastic materials more than any other, and he will not quarrel with its ranking.

The book ends with a chapter on finite strain and a chapter on the finite deformation of elastic and hyperelastic materials. The final chapter, Chapter X, treats a subject in which the reviewer has had a special interest and which therefore occasions that urge to haggle mentioned above. The chapter opens with the following definition of an elastic material: 'A material will be called elastic, if it possesses a homogeneous stress-free natural state, and if in an appropriately defined finite neighborhood of this state there exists a one-to-one correspondence between the Eulerian stress tensor **T** and Almansi's strain tensor **U**.' Now this simply is an inadequate definition of an elastic material, for it excludes all but isotropic materials. In a perfectly elastic material of less symmetry, it is just impossible to express the Eulerian stress tensor as a function of Almansi's strain tensor only; i.e. in general, $T_{ij} = T_{ij}(\mathrm{U})$ only for isotropic elastic materials. In a material of general symmetry, the Eulerian stress tensor is always a function of the deformation gradient, but it occurs in combinations other than that appearing in **U** unless the material is isotropic. What Prof. Prager calls Kirchhoff's stress tensor and denotes by $\bar{S}_{ij}$ (cf. equation (4.6), Ch. IX) is, in a perfectly elastic material, always a function of Green's strain tensor $\bar{U}_{ij}$ (cf. equation (2.7), Ch. IX), provided that the stress enjoys the proper invariance. Though the stress in a perfect fluid also has this property, one can take the invertibility of the relations $\bar{S}_{ij} = \bar{S}_{ij}(\mathrm{U})$ as a possible definition of a solid elastic material.

The final section of Chapter X treats the problem of uniqueness and stability of elastic deformations. An attempt is made to derive Euler's criterion for the buckling of a column of isotropic elastic material loaded on its ends. Though a noble objective, the derivation suffers logical gaps and errors. For example, equation (4.20) assumes that the equilibrium, finitely deformed state of the column, the stability of which is under investigation, is isotropic in the sense that small deformation about that state is related to a small increment in the stress by the usual isotropic formula. This does not seem to be justified. But there are so many other special assumptions regarding the nature of the material and approximations made in the ensuing argument that one could hardly have confidence in Euler's result if it were not known beforehand.

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Boundary Layer Theory (fourth edition). By H. SCHLICHTING. New York: McGraw-Hill, 1960. 647 pp. £6. 8*s*.

This second English edition of Professor Schlichting's well-known book is basically a translation of the third edition, published in German in 1958. The translator is again Professor Kestin, and he must again be congratulated for producing a very readable translation. The result is a welcome change from the large number of excessively literal translations of technical books (particularly from the German language) which have been published in recent years.

In content, this edition differs appreciably from the first English edition. The changes arise mainly from the inclusion of new material rather than from a revision of the old. For the most part, the additions represent an account of original papers published since the first edition of the book, and it is not surprising to find that they occur mainly in the chapters concerned with compressible flow and turbulent boundary layers.

The new edition, like its predecessors, represents the very best in the German *Handbuch* tradition. It is up-to-date, comprehensive, and integrated into a homogeneous whole in a way which could be achieved only by a single author. The theoretical physicist may not always find individual points of fundamental interpretation for which he is looking, but both he and the engineer will find virtually every established result in boundary-layer theory. The book remains pre-eminent in its field, and is indispensable to anyone who is interested in boundary layers.

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